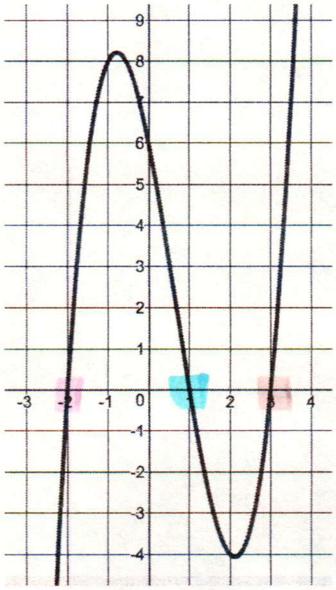
AGS 3 Name: ______________________ Period: _____ Date: ___________

Notes 3.8 – Finding Polynomial Roots

Warmup – Write the equation in factored form for each given graph.

1.

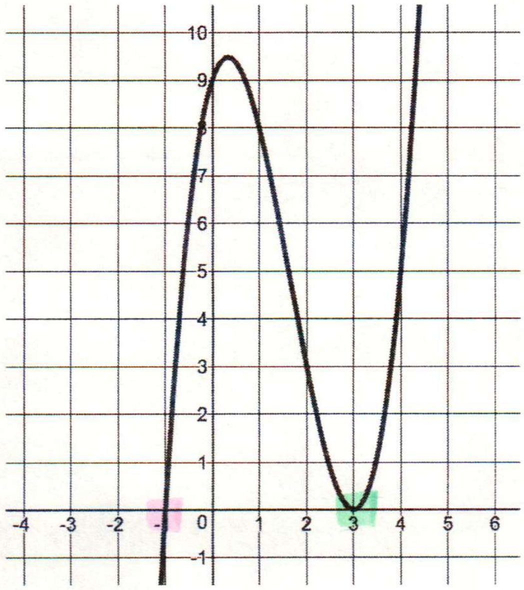


Roots: $x = -2, 1, 3$

Equation:

$y = (x+2)(x-1)(x-3)$

2.



Roots: $x = -1, 3$

Equation:

$y = (x+1)(x-3)(x-3)$

or

$y = (x+1)(x-3)^2$

Investigation

Think about how you find the prime factorization of a number.

Find the prime factorization of 120. Hint: make a factor tree

120 < 30, 4; 30 < 6, 5; 6 < 2, 3; 4 < 2, 2

$2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$

When we make a factor tree, we find one number we believe goes into the number. We divide by that number and if it goes in evenly, then it is a factor. Then we repeat the process until all remaining numbers are prime.

120 ← even so $120 \div 2 = 60$

2 60 ← even so $60 \div 2 = 30$

2 30 ← even so $30 \div 2 = 15$

2 15 ← multiple of 5 so $15 \div 5 = 3$

3 5

repeated division to break down to factors

$2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$

Finding the roots of a polynomial uses the same process. Repeatedly divide (or factor) until you only have factors that look like $(x \pm number)$ left.

$f(x) = x^3 + 3x^2 - 4x - 12$ Given factor: $(x + 3)$

Divide (Long or synthetic):

$$\begin{array}{r|rrrr} -3 & 1 & 3 & -4 & -12 \\ & \downarrow & -3 & 0 & 12 \\ \hline & 1 & 0 & -4 & 0 \\ & x^2 & x & c & r \end{array}$$

$x^2 - 4$

List Roots:

$x = -3$ ←

$x = -2$ ←

$x = 2$ ←

Root → factor

$x = -3$
$+3 \quad +3$

$x + 3 = 0$

$(x+3)$

Divide again or Factor or Quadratic Formula:

$x^2 - 4 \Rightarrow (x+2)(x-2)$

Write the equation in factored form:

$y = (x+3)(x+2)(x-2)$

Practice, find the roots, then write the equation in factored form.

a. $f(x) = x^3 + 6x^2 + 11x + 6$ Given factor: $(x + 1)$ ← Since we know it is a factor, there should not be a remainder

$$\begin{array}{r|rrrr} -1 & 1 & 6 & 11 & 6 \\ & & -1 & -5 & -6 \\ \hline & 1 & 5 & 6 & 0 \end{array}$$

$x^2 + 5x + 6$

$(x+2)(x+3)$

Roots

$x = -1, -2, -3$

$$\boxed{y = (x+1)(x+2)(x+3)}$$

b. $f(x) = x^3 - 5x^2 - 3x + 15$ Given factor: $(x - 5)$

Roots

$x = 5, \sqrt{3}, -\sqrt{3}$

$$\begin{array}{r|rrrr} 5 & 1 & -5 & -3 & 15 \\ & & 5 & 0 & -15 \\ \hline & 1 & 0 & -3 & 0 \end{array}$$

$x^2 - 3$

$$\boxed{y = (x-5)(x+\sqrt{3})(x-\sqrt{3})}$$

$x^2 - 3 = 0$

$x^2 = 3$

$x = \pm\sqrt{3}$

Practice, find the roots, then write the equation in factored form.

c. $f(x) = x^3 + 3x^2 - 12x - 18$ Given factor: $(x - 3)$

$$\begin{array}{r|rrrr} 3 & 1 & 3 & -12 & -18 \\ & & 3 & 18 & 18 \\ \hline & 1 & 6 & 6 & 0 \end{array}$$

$x^2 + 6x + 6$

$$x = \frac{-6 \pm \sqrt{(6)^2 - 4(1)(6)}}{2(1)}$$

$$x = \frac{-6 \pm \sqrt{12}}{2} \qquad x = -3 \pm \sqrt{3}$$

Roots

$x = 3$

$x = -3 + \sqrt{3}$

$x = -3 - \sqrt{3}$

$y = (x-3)(x-(-3+\sqrt{3}))(x-(-3-\sqrt{3}))$

$\boxed{y = (x-3)(x+3-\sqrt{3})(x+3+\sqrt{3})}$

d. $f(x) = x^4 - 16$ Given factor: $(x - 2)$

$(x^2+4)(x^2-4)$

$(x^2+4)(x+2)(x-2)$

$x^2 + 4 = 0$

$x^2 = -4$

$x = \pm\sqrt{-4}$

$x = \pm 2i$

Roots

$x = -2$

$x = 2$

$x = 2i$

$x = -2i$

$\boxed{y = (x+2)(x-2)(x+2i)(x-2i)}$